A, and the square root of $F$, where $F$ is the family size, and inversely proportional to the number of inches of rain, $R$.

In one month, the average daily temperature is 78°F and the number of inches of rain is 5.6. If the average family of four who has a thousand square feet of lawn pays $72.00 for water for that month, estimate the water bill in the same month for the average family of six who has 1500 ft$^2$ of lawn.

### 4.2 LINEAR INEQUALITIES

Suppose your campus bookstore needs to determine how many textbooks for a particular course must be sold so that the bookstore's textbook revenue is greater than its cost. To determine this number, an inequality would be set up. In this section, we will discuss how to set up and how to solve an inequality.

The symbols of inequality are as follows.

**SYMBOLS OF INEQUALITY**

- $a < b$ means that $a$ is less than $b$.
- $a \leq b$ means that $a$ is less than or equal to $b$.
- $a > b$ means that $a$ is greater than $b$.
- $a \geq b$ means that $a$ is greater than or equal to $b$.

An inequality consists of two (or more) expressions joined by an inequality sign.

**Examples of inequalities**

$$3 < 5, \quad x < 2, \quad 3x - 2 \geq 5$$

A statement of inequality can be used to indicate a set of real numbers. For example, $x < 2$ represents the set of all real numbers less than 2. Listing all these numbers is impossible, but some are $-2, -1.234, -1, -\frac{1}{2}, 0, \frac{97}{163}, 1$.

To indicate all real numbers less than 2, we can use the number line. The number line was discussed in Chapter 1.

#### Solving Inequalities

To indicate the solution set of $x < 2$ on the number line, we draw an open circle at 2 and a line to the left of 2 with an arrow at its end. This technique indicates that all points to the left of 2 are part of the solution set. The open circle indicates that the solution set does not include the number 2.

![Number line showing $x < 2$]

To indicate the solution set of $x \leq 2$ on the number line, we draw a closed (or darkened) circle at 2 and a line to the left of 2 with an arrow at its end. The closed circle indicates that the 2 is part of the solution.
EXAMPLE 1 \(\text{Graphing a Less Than or Equal to Inequality}\)

Graph the solution set of \(x \leq -1\), where \(x\) is a real number, on the number line.

**SOLUTION** The numbers less than or equal to \(-1\) are all the points on the number line to the left of \(-1\) and \(-1\) itself. The closed circle at \(-1\) shows that \(-1\) is included in the solution set.

![Graph of \(x \leq -1\)](image)

The inequality statements \(x < 2\) and \(2 > x\) have the same meaning. Note that the inequality symbol points to the \(x\) in both cases. Thus, one inequality may be written in place of the other. Likewise, \(x > 2\) and \(2 < x\) have the same meaning. Note that the inequality symbol points to the \(2\) in both cases. We make use of this fact in Example 2.

EXAMPLE 2 \(\text{Graphing a Less Than Inequality}\)

Graph the solution set of \(3 < x\), where \(x\) is a real number, on the number line.

**SOLUTION** We can restate \(3 < x\) as \(x > 3\). Both statements have identical solutions. Any number that is greater than 3 satisfies the inequality \(x > 3\). The graph includes all the points to the right of 3 on the number line. To indicate that 3 is not part of the solution set, we place an open circle at 3.

![Graph of \(3 < x\)](image)

We can find the solution to an inequality by adding, subtracting, multiplying, or dividing both sides of the inequality by the same number or expression. We use the procedure discussed in Section 3.2 to isolate the variable, with one important exception: When both sides of an inequality are multiplied or divided by a negative number, the direction of the inequality symbol is reversed.

EXAMPLE 3 \(\text{Multiplying by a Negative Number}\)

Solve the inequality \(-x > 3\) and graph the solution set on the number line.

**SOLUTION** To solve this inequality, we must eliminate the negative sign in front of the \(x\). To do so, we multiply both sides of the inequality by \(-1\) and change the direction of the inequality symbol.

\[-x > 3\]
\[-1(-x) < -1(3)\] Multiply both sides of the inequality by \(-1\) and change the direction of the inequality symbol.

\[x < -3\]

The solution set is graphed on the number line as follows.

![Graph of \(x < -3\)](image)
**EXAMPLE 4** *Dividing by a Negative Number*

Solve the inequality $-5x < 20$ and graph the solution set on the number line.

**SOLUTION** Solving the inequality requires making the coefficient of the $x$ term 1. To do so, divide both sides of the inequality by $-5$ and change the direction of the inequality symbol.

$$
\begin{align*}
-5x &< 20 \\
\frac{-5x}{-5} &> \frac{20}{-5} \\
x &> -4
\end{align*}
$$

The solution set is graphed on the number line as follows.

**EXAMPLE 5** *Solving an Inequality*

Solve the inequality $3x - 5 > 13$ and graph the solution set on the number line.

**SOLUTION** To find the solution set, isolate $x$ on one side of the inequality symbol.

$$
\begin{align*}
3x - 5 &> 13 \\
3x - 5 + 5 &> 13 + 5 \\
3x &> 18 \\
\frac{3x}{3} &> \frac{18}{3} \\
x &> 6
\end{align*}
$$

Thus, the solution set to $3x - 5 > 13$ is all real numbers greater than 6.

Note that in Example 5, the *direction of the inequality symbol* did not change when both sides of the inequality were divided by the positive number 3.

**EXAMPLE 6** *A Solution of Only Integers*

Solve the inequality $x + 5 < 9$, where $x$ is an integer, and graph the solution set on the number line.

**SOLUTION** To find the solution set, isolate $x$ on one side of the inequality symbol.

$$
\begin{align*}
x + 5 &< 9 \\
x + 5 - 5 &< 9 - 5 \\
x &< 4
\end{align*}
$$

Since $x$ is an integer and is less than 4, the solution set is the set of integers less than 4, or $\{\ldots, -2, -1, 0, 1, 2, 3\}$. To graph the solution set, we make solid dots at

---

**DID YOU KNOW?**

*What a Bore*

In the production of machine parts, engineers must allow a certain tolerance in the way parts fit together. For example, the boring machines that grind cylindrical openings in an automobile's engine block must create a cylinder that allows the piston to move freely up and down, but still fit tightly enough to ensure that compression and combustion are complete. The allowable tolerance between parts can be expressed as an inequality. For example, the diameter of a cylinder may need to be no less than 3.383 in. and no greater than 3.387 in. We can represent the allowable tolerance as $3.383 \leq t \leq 3.387$. 

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the corresponding points on the number line. The ellipsis (the three dots) to the left of \(-3\) indicate that all the integers to the left of \(-3\) are included.

-5 –4 –3 –2 –1 0 1 2 3 4 5

\(x < 4, \quad x \text{ an integer}\)

**TIMELY TIP** Some linear inequalities have no solution. When solving an inequality, if you obtain a statement that is always false, such as \(3 > 5\), the answer is *no solution*. There is no real number that makes the statement true.

Some linear inequalities have all real numbers as their solution. When solving an inequality, if you obtain a statement that is always true, such as \(2 > 1\), the solution is *all real numbers*.

## Solving Compound Inequalities

An inequality of the form \(a < x < b\) is called a *compound inequality*. Consider the compound inequality \(-3 < x \leq 2\), which means that \(-3 < x\ and \(x \leq 2\).

### EXAMPLE 7 A Compound Inequality

Graph the solution set of the inequality \(-3 < x \leq 2\)

a) where \(x\) is an integer.  b) where \(x\) is a real number.

**SOLUTION**

a) The solution set is all the integers between \(-3\) and \(2\), including the \(2\) but not including the \(-3\), or \(\{-2, -1, 0, 1, 2\}\).

-5 –4 –3 –2 –1 0 1 2 3 4 5

\(-3 < x \leq 2, \quad x \text{ an integer}\)

b) The solution set consists of all the real numbers between \(-3\) and \(2\), including the \(2\) but not including the \(-3\).

-5 –4 –3 –2 –1 0 1 2 3 4 5

\(-3 < x \leq 2\)

### EXAMPLE 8 Solving a Compound Inequality

Solve the compound inequality for \(x\) and graph the solution set.

\(-4 < \frac{x + 3}{2} \leq 5\)

**SOLUTION** To solve this compound inequality, we must isolate the \(x\) in the middle term. To do so, we use the same principles used to solve inequalities.

\(-4 < \frac{x + 3}{2} \leq 5\)

"In mathematics the art of posing problems is easier than that of solving them."

Georg Cantor
Multiply each part of the inequality by 2.

\[ 2(-4) < \frac{2(x + 3)}{2} \leq 2(5) \]

\[ -8 < x + 3 \leq 10 \]

Subtract 3 from each part of the inequality.

\[ -8 - 3 < x + 3 - 3 \leq 10 - 3 \]

\[ -11 < x \leq 7 \]

The solution set is graphed on the number line as follows.

---

**EXAMPLE 9 Average Grade**

A student must have an average (the mean) on five tests that is greater than or equal to 80% but less than 90% to receive a final grade of B. Devon’s grades on the first four tests were 98%, 76%, 86%, and 92%. What range of grades on the fifth test would give him a B in the course?

**SOLUTION** The unknown quantity is the range of grades on the fifth test. First construct an inequality that can be used to find the range of grades on the fifth test. The average (mean) is found by adding the grades and dividing the sum by the number of tests.

Let \( x = \) the fifth grade. Then

\[ \text{Average} = \frac{98 + 76 + 86 + 92 + x}{5} \]

For Devon to obtain a B in this course, his average must be greater than or equal to 80 but less than 90.

\[ 80 \leq \frac{98 + 76 + 86 + 92 + x}{5} < 90 \]

\[ 80 \leq \frac{352 + x}{5} < 90 \]

\[ 5(80) \leq (\frac{352 + x}{5}) < 5(90) \]

Multiply the three terms of the inequality by 5.

\[ 400 \leq 352 + x < 450 \]

Subtract 352 from all three terms.

\[ 400 - 352 \leq 352 - 352 + x < 450 - 352 \]

\[ 48 \leq x < 98 \]

Thus, a grade of 48% up to but not including a grade of 98% on the fifth test will result in a grade of B in this course.

---

**TIMELY TIP** Remember to change the direction of the inequality symbol when multiplying or dividing both sides of an inequality by a negative number.
SECTION 4.2  EXERCISES

CONCEPT/Writing Exercises

1. Give the four inequality symbols we use in this section and indicate how each is read.

2. a) What is an inequality?
   b) Give an example of three inequalities.

3. When solving an inequality, under what conditions do you need to change the direction of the inequality symbol?

4. Does $x < 2$ have the same meaning as $2 > x$? Explain.

5. Does $x > -3$ have the same meaning as $-3 < x$? Explain.

6. When graphing the solution set to an inequality on the number line, when should you use an open circle and when should you use a closed circle?

7. a) What is a compound inequality?
   b) Give an example of a compound inequality.

8. a) Give an example of an inequality whose solution on a number line would contain a closed circle.
   b) Give an example of an inequality whose solution on a number line would contain an open circle.

Practice the Skills

In Exercises 9–26, graph the solution set of the inequality, where $x$ is a real number, on the number line.

9. $x \geq 4$
10. $x < 7$
11. $x - 8 \geq 3$
12. $x + 5 > -2$
13. $-3x \leq 18$
14. $-4x < 12$
15. $\frac{x}{5} < 4$
16. $\frac{x}{6} > 2$
17. $\frac{-x}{3} \geq 3$
18. $\frac{x}{2} \geq -4$
19. $2x + 6 \equiv 14$
20. $3x + 12 < 5x + 14$
21. $4(2x - 1) < 2(4x - 3)$
22. $-2(x - 3) \leq 4x + 6$
23. $-1 \leq x \leq 3$
24. $-4 < x < 1$
25. $3 < x - 7 \leq 6$
26. $\frac{1}{2} < \frac{x + 4}{2} \leq 4$

In Exercises 27–46, graph the solution set of the inequality, where $x$ is an integer, on the number line.

27. $x > 1$
28. $-2 < x$
29. $-3x \leq 27$
30. $3x \geq 27$
31. $x - 2 < 4$
32. $-5x \leq 15$
33. $\frac{x}{3} \leq -2$
34. $\frac{x}{4} \geq -3$
35. $\frac{-x}{4} \geq 2$
36. $\frac{3x}{5} \leq 3$
37. $-15 < -4x - 3$
38. $-5x - 1 < 17 - 2x$
39. $3(x + 4) \equiv 4x + 13$
40. $-2(x - 1) < 3(x - 4) + 5$
41. $5(x + 4) - 6 \leq 2x + 8$
42. $-3 \leq x < 5$
43. $1 > -x > -5$
44. $-2 < 2x + 3 < 6$
45. $0.3 \leq \frac{x + 2}{10} \leq 0.5$
46. $-\frac{1}{3} < \frac{x - 2}{12} \leq \frac{1}{4}$

Problem Solving

47. Federal Drug Costs The following bar graph shows the government's projected cost, in billions of dollars, for the Medicare prescription drug benefit for the years 2005–2015.

![Projected Cost of Medicare Prescription Drug Benefit](Source: CMS)
In which years is the projected cost

a) > $60 billion?  b) ≤ $60 billion?

c) > $20 billion?  d) ≤ $40 billion?

48. Work Stoppages The following bar graph shows the number of work stoppages involving at least 1000 workers, including strikes and lockouts, in the United States for the years 1999–2004.

![Bar graph of work stoppages in the U.S.]


In which years was the number of work stoppages in the United States

a) <25?  b) >30?  c) <40?  d) >20?

49. Video Rental Movie Mania offers two rental plans. One has an annual fee, and the other has no annual fee. The annual membership fee and the daily charge per video for each plan are shown in the table. Determine the maximum number of videos that can be rented for the no fee plan to cost less than the annual fee plan.

<table>
<thead>
<tr>
<th>Rental Plan</th>
<th>Yearly Fee</th>
<th>Daily Charge per Video</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual fee</td>
<td>$30</td>
<td>$1.49</td>
</tr>
<tr>
<td>No fee</td>
<td>None</td>
<td>$2.99</td>
</tr>
</tbody>
</table>

50. Salary Plans Bobby Exler recently accepted a sales position in Portland, Oregon. He can select between the two salary plans shown in the table. Determine the dollar amount of weekly sales that would result in Bobby earning more with Plan B than with Plan A.

<table>
<thead>
<tr>
<th>Salary Plan</th>
<th>Weekly Salary</th>
<th>Commission on Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plan A</td>
<td>$500</td>
<td>6%</td>
</tr>
<tr>
<td>Plan B</td>
<td>$400</td>
<td>8%</td>
</tr>
</tbody>
</table>

51. College Employment To be eligible for financial assistance for college, Samantha Lavin can earn no more than $3000 during her summer employment. She will earn $725 cutting lawns. She is also earning $6.50 per hour at an evening job as a cashier at a grocery store. What is the maximum number of hours Samantha can work at the grocery store without jeopardizing her financial assistance?

52. Moving Boxes A janitor must move a large shipment of books from the first floor to the fifth floor. Each box of books weighs 60 lb, and the janitor weighs 180 lb. The sign on the elevator reads, “Maximum weight 1200 lb.”

a) Write a statement of inequality to determine the maximum number of boxes of books the janitor can place on the elevator at one time. (The janitor must ride in the elevator with the books.)

b) Determine the maximum number of boxes that can be moved in one trip.

53. Parking Costs A parking garage in Richmond, Virginia, charges $1.75 for the first hour and $0.50 for each additional hour or part of an hour. What is the maximum length of time Tom Shalhoub can park in the garage if he wishes to pay no more than $4.25?

54. Making a Profit For a business to realize a profit, its revenue, \( R \), must be greater than its costs, \( C \); that is, a profit will only result if \( R > C \) (the company breaks even when \( R = C \)). A book publishing company has a weekly cost equation of \( C = 2x + 2000 \) and a weekly revenue equation of \( R = 12x \), where \( x \) is the number of books produced and sold in a week. How many books must be sold weekly for the company to make a profit?

55. Finding Velocity The velocity, \( v \), in feet per second, \( t \) sec after a tennis ball is projected directly upward is given by the formula \( v = 84 - 32t \). How many seconds after being projected upward will the velocity be between 36 ft/sec and 68 ft/sec?

56. Speed Limit The minimum speed for vehicles on a highway is 40 mph, and the maximum speed is 55 mph. If Philip Rowe has been driving nonstop along the highway for 4 hr, what range in miles could he have legally traveled?
57. A Grade of B In Example 9 of this section, what range of grades on the fifth test would result in Devon receiving a grade of B if his grades on the first four tests were 78%, 64%, 88%, and 76%?

58. Tent Rental The Cuyahoga Community College Planning Committee wants to rent tents for the spring job fair. Rent-a-Tent charges $325 for setup and delivery of its tents. This fee is charged regardless of the number of tents delivered and set up. In addition, Rent-a-Tent charges $125 for each tent rented. If the minimum amount the planning committee wishes to spend is $950 and the maximum amount they wish to spend is $1200, determine the minimum and maximum number of tents the committee can rent.

CHALLENGE PROBLEMS/GROUP ACTIVITIES

59. Painting a House Donovan Davis is painting the exterior of his house. The instructions on the paint can indicate that 1 gal covers from 250 to 400 ft². The total surface of the house to be painted is 2750 ft². Determine the number of gallons of paint he could use and express the answer as an inequality.

60. Final Exam Teresa’s five test grades for the semester are 86%, 78%, 68%, 92%, and 72%. Her final exam counts one-third of her final grade. What range of grades on her final exam would result in Teresa receiving a final grade of B in the course? (See Example 9.)

61. A student multiplied both sides of the inequality \(-\frac{1}{2} x = 4\) by \(-3\) and forgot to reverse the direction of the inequality symbol. What is the relation between the student’s incorrect solution set and the correct solution set? Is there any number in both the correct solution set and the student’s incorrect solution set? If so, what is it?

INTERNET/RESEARCH ACTIVITY

62. Find a newspaper or a magazine article that contains the mathematical concept of inequality.

a) From the information in the article write a statement of inequality.

b) Summarize the article and explain how you arrived at the inequality statement in part (a).

4.3 GRAPHING LINEAR EQUATIONS

The profit, \(p\), of a company may depend on the amount of sales, \(s\). The cost, \(c\), of mailing a package may depend on the weight, \(w\), of the package. Real-life problems, such as these two examples, often involve two or more variables. In this section, we will discuss how to graph equations with two variables.

To be able to work with equations with two variables requires understanding the Cartesian (or rectangular) coordinate system, named after the French mathematician René Descartes. The rectangular coordinate system consists of two perpendicular number lines (Fig. 4.1). The horizontal line is the \(x\)-axis, and the vertical line is the \(y\)-axis. The point of intersection of the \(x\)-axis and \(y\)-axis is called the origin. The numbers on the axes to the right and above the origin are positive. The numbers on the axes on the left and below the origin are negative. The axes divide the plane into four parts: the first, second, third, and fourth quadrants.